An Efficient Identification and Implementation of Preisach-Stoner-Wohlfarth Vector Hysteresis Model

Lei Liu, W. N. Fu

Department of Electrical Engineering, The Hong Kong Polytechnic University, Hong Kong
eewnfu@polyu.edu.hk

The Preisach-Stoner-Wohlfarth vector hysteresis model is endowed with many desirable properties, but its application is restricted due to its slow computation. Furthermore, the identification of this model involves artwork rather than systematic treatment. This paper proposes a fast and efficient approximation to Stoner-Wohlfarth hysteron, which prevents the direct evaluation of nonlinear equations and the expensive evaluation of the Preisach diagram. The identification problem is formulated as a classical nonnegative least square regression. This leads to a robust identification.

***Index Terms*— nonnegative least squares, Preisach diagram, Stoner-Wohlfarth model, vector hysteresis**

I. Introduction

PHENOMENOLOGICAL MODELLING of vector hysteresis has long been centered on the Preisach model and the Stoner-Wohlfarth (SW) model. The former is supported by its simulation accuracy owing to its sophisticated identification and implementation, the latter has some features of physical realities and used extensively in the area of magnetic recording. As a result, there is a tendency toward combing these models, which results in the Preisach-Stoner-Wohlfarth (PSW) model. In this type of models, single-domain, uniaxial magnetic entities are described by the independently SW model. The interactions between them are accounted for through the employment of the Preisach diagram. This modification eliminates the intrinsic flaws of the SW model, for instance, the failure in predicting non-symmetrical minor loops, while all the important merits, such as rotational loss properties, are naturally kept. However, this generalization is computationally heavy. The behavior of each hysteron strongly depends on its coercive and interaction fields, as well as the orientation of its easy axis. To predict the state of a SW hysteron, the solutions of quartic equations, associated with shifted asteroid with different sizes, are required. Furthermore, the distribution function of the PSW model is generally unknown. The identification from limited experimental data is conducted in an ad hoc manner.

To circumvent the aforementioned limitations of the PSW model, in this paper, a simplified SW hysteron is employed to accelerate the computation and a constrained linear programming is proposed to address the identification of the PSW model.

II. Model Description

A. *Simplified Stoner-Wohlfarth hysteron*

The free energy of a SW hysteron subject to an magnetic field $\boldsymbol{H}$ is

$$G = \frac{1}{2} M_s H_{sw} \sin^2 \theta - M_s H_{per} \sin\theta - M_s H_{par} \cos\theta \tag{1}$$

where M_s is the saturation magnetization of the SW hysteron. H_{sw} is the switching field depending on the anisotropy of the material. H_{per} and H_{par} are the field components perpendicular and parallel to the easy axis. The angle between the particle axis and the magnetic moment of the SW hysteron is θ. The magnetization process of the SW hysteron is governed by the stability properties of (1). By imposing the following condition

$$\partial G / \partial \theta = 0, \tag{2.a}$$

$$\partial^2 G / \partial \theta^2 = 0, \tag{2.b}$$

the bifurcation set of the SW hysteron is determined. The curve represented by (2) when θ varies in [-π,π] is known as the astroid, as shown in Fig. 1.

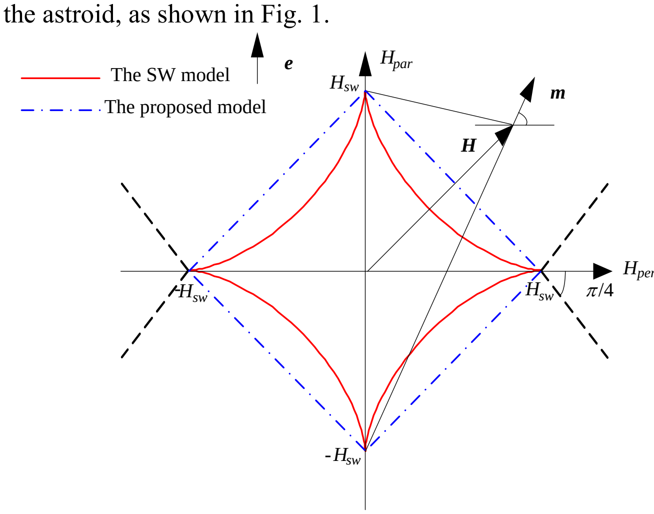

Fig. 1. The astroid curve (solid line) and square curve (dashed dot-line) and some solution of simplified SW hysteron.

The solving of (2.*a*) is iterative. This drawback slows down the evaluation of PSW model. Based on [1], an expedient modification of the SW equations is proposed. The angular dependence of this simplified model is fairly agreed with the SW model. The magnetization of the simplified SW hysteron is demonstrated in the following way: let the orientation of the easy axis be on the H_{per}-H_{par} plane, which is denoted as $\boldsymbol{e}$. The dashed lines in Fig. 1 divide the H_{per}-H_{par} plane into the left, center and right regions, respectively. For the left- and right-regions, the orientation of the magnetization $\boldsymbol{m}$ is determined by

$$\boldsymbol{m} = (\boldsymbol{H} \pm H_{sw} \boldsymbol{k} \times \boldsymbol{e}) / \left| (\boldsymbol{H} \pm H_{sw} \boldsymbol{k} \times \boldsymbol{e}) \right| \tag{3}$$

where $\boldsymbol{k}$ is the unit vector normal to the H_{per}-H_{par} plane. + (−) is used for the right- (left-) regions, respectively. For the center- region, the result is

$$\boldsymbol{m} = (\boldsymbol{H} \pm H_{sw}\boldsymbol{e}) / |(\boldsymbol{H} \pm H_{sw}\boldsymbol{e})| \tag{4}$$

Where the sign is the same as $\boldsymbol{H}\cdot\boldsymbol{e}$ when the tip of $\boldsymbol{H}$ is outside the square, otherwise it is the same as $\boldsymbol{m}\cdot\boldsymbol{e}$ when the tip of H is inside the square, which is the origin of the hysteresis of the PSW model. Here the original bifurcation set is replace by the square in the proposed PSW model. This simplification not only provides similar behavior of the SW hysteron but also simplify the description of the Preisach diagram as is detailed in the full paper.

B. Model implementation

Let us consider a general SW hysteron which has its own easy axis and switching field H_{sw}. To introduce the interaction due to its neighboring hysteron, H_{par} in (1) is replaced by $H_{par} - H_i$, i.e., the orientation of H_i is parallel to the easy axis. Given n different easy axes θ_{ei}, i = 1, …, nd, used in the model, nd Preisach diagram are defined, as shown in Fig. 2, for every easy axis. Each Preisach diagram depicts the distribution of H_{sw} and H_i of SW hysterons. As a consequence, the component of the total magnetization due to all the SW hysterons that have easy axis θ_{ei} is given as

$$\boldsymbol{M}_{\theta_{ei}} = \iint_{P_i} \boldsymbol{S}_{\theta_{ei}}(H_i, H_{sw}, H_x, H_y)\rho(H_i, H_{sw})dH_i dH_{sw} \tag{5}$$

and the total magnetization is obtained:

$$\boldsymbol{M} = \sum_{i=1}^{nd} w_i \boldsymbol{M}_{\theta_{ei}} \tag{6}$$

where w_i represents the distribution density of the easy axes. In the engineering viewpoint, only one easy axis is sufficient to model anisotropic material, while uniform distribution is usually assumed for isotropic material.

C. Model identification

The identification problem of the PSW model, at this stage, is posed as the finding of the density function $\rho(H_i, H_{sw})$. To achieve this goal, the domain of the Preisach diagram is discretized [2], as shown in Fig. 2, where H_s is the saturation magnetic field intensity. The classical triangulation is applied to the domain. The largest triangle size is determined by the incremental change of the applied magnetic field intensity. It is noted that only half of the domain in Fig. 2 is required to describe the Preisach diagram since the domain is symmetric with respect to the H_{sw}-axis. Besides the reduction of computational burden, this method removes the numerical discrepancies that breaks the intrinsic symmetry of the density function $\rho(H_i, H_{sw})$.

Let the density function $\rho(H_i, H_{sw})$ on each cell of the partitioned domain be uniform, then the identification problem becomes finding the vector $\boldsymbol{R} = (\rho_k)_{mt\times1}$, where mt is the number of cells. If there are a series experimental data (H_{xtj}, H_{ytj}) and (M_{xtj}, M_{ytj}), j = 1,…, nt. The substitution of $\boldsymbol{R}$ and these experimental data into (5) and (6) gives a linear system

$$\boldsymbol{M} = \boldsymbol{SS} \cdot \boldsymbol{R} \tag{7}$$

where $\boldsymbol{M} = [\boldsymbol{M}_x, \boldsymbol{M}_y]^{\mathrm{T}}$, and $\boldsymbol{M}_x = (M_{xtj})_{nt\times1}$, $\boldsymbol{M}_y = (M_{ytj})_{nt\times1}$; and

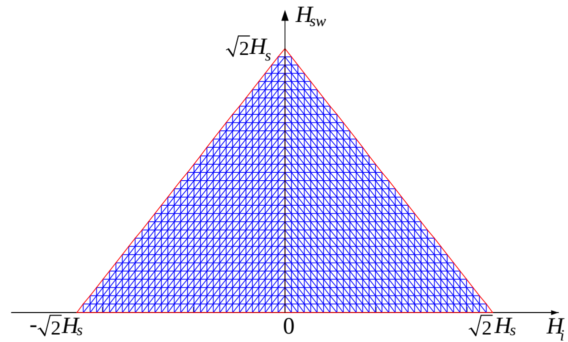

Fig. 2. Partitioned domain of the Preisach diagram

$$\boldsymbol{SS} = (SS_{j,k})_{nt\times ne}$$
$$= (\sum_{i=1}^{nd} w_i \iint_{\Omega_k} \boldsymbol{S}_{\theta_{ei}}(H_i, H_{sw}, H_{xtj}, H_{ytj})dH_i dH_{sw})_{nt\times ne}$$

For anisotropic material, $(w_i)_{1\times nd} = [1, O_{1\times(nd-1)}]^{\mathrm{T}}$; for isotropic material , $(w_i)_{1\times nd} = [1/nd, \ldots, 1/nd]^{\mathrm{T}}$. In addition, (7) is constrained by

$$\sum_{i=1}^{ne} \rho_i = 1, \qquad \rho_i \geq 0 \tag{8}$$

The solving of (7) and (8) is equivalent to a quadratic programing problem with a convex feasible set. There is a standard solver for this type of problem [3], which could approach a reasonable approximation at a reasonable number of iterations.

III. Results

A major hysteresis loop is used to identify the scalar Preisach model and the PSW model, respectively. As a validated scalar hysteresis model, the Preisach model is used to verify the effectiveness of the proposed simplified PSW and its parameter identification. As shown in Fig. 3, they are matched fairly well. As for more results, especially those associated with vector hysteresis properties, they will be detailed in the full paper.

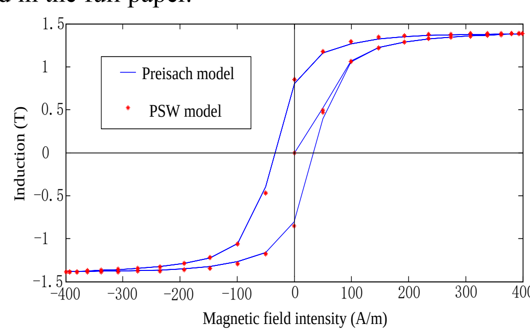

Fig. 3. The comparison between Preisach model and the proposed PSW model.

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